METO 611 MIDTERM II EXAMINATION

This is a closed book 50 minute examination covering the second five weeks of Meto611. There are two problems, each worth 50 points. Please look over the whole examination before beginning. Please check to make sure your answers make sense.

1a. Descriptive dynamics Please characterize the following observed phenomena briefly

<table>
<thead>
<tr>
<th>Name</th>
<th>Where found?*</th>
<th>Time-scale?¹</th>
<th>Phase propagation direction?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-biennial Oscillation</td>
<td>Lower tropical stratosphere</td>
<td>~2 years</td>
<td>downward</td>
</tr>
<tr>
<td>Madden-Julian Oscillation (a.k.a. 30-60 day oscillation)</td>
<td>Tropical troposphere</td>
<td>30-60 days</td>
<td>eastward</td>
</tr>
<tr>
<td>Yanai Wave (a.k.a. mixed Rossby-Gravity Wave)</td>
<td>Upper ocean, lower stratosphere</td>
<td>Varies, e.g. 20dy ocean</td>
<td>Depends: eastward at high frequency, westward at low freq.</td>
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</tbody>
</table>

*stratosphere, troposphere, ocean, tropics, polar latitudes
¹Days, weeks, months, years?
1.b) In a single paragraph please briefly define ‘geostrophic adjustment’ and explain the importance of geostrophic adjustment for numerical weather prediction. Be sure to mention the radius of deformation in your discussion.

Geostrophic adjustment refers to the process by which a fluid, which initially has mass and momentum fields that do not satisfy the geostrophic equations, are gradually brought into geostrophic equilibrium. Geostrophic adjustment is a key issue in numerical weather prediction, because initial conditions for forecast models, if simply specified from observations, will not satisfy the geostrophic relationship. What will happen in the model is geostrophic adjustment in which gravity waves will be generated and the flow will adjust (the mass field adjusting to the momentum field on scales less than the radius of deformation and the momentum field adjusting to the mass field on scales greater than the radius of deformation).

1.c) Suppose that at some initial time a pressure anomaly that is independent of height, elongated in the zonal direction, and antisymmetric about the equator is introduced, e.g.

\[
p = \begin{cases} 
p_o \sin(\pi y / R_d) & -R_d < y < R_d \\
0 & \text{otherwise}
\end{cases} \quad 0 < x < 20R_d
\]

The pressure anomaly is confined within an equatorial radius of deformation, \(R_d\), of the equator. Based on your knowledge of equatorial waves, what waves will this initial disturbance excite, and direction will they propagate. Try to think about which waves have the same symmetries as the forcing.

We need to think about those waves on the equatorial dispersion diagram at small zonal wavenumber (\(k\) small) that are antisymmetric in pressure (thus symmetric in \(v\)) and whose meridional structure in pressure is confined to within a radius of deformation of the equator. We find a low \(n\) gravity wave, the Yanai wave, and perhaps the first Rossby wave.

[Diagram showing gravity waves, Kelvin wave, Yanai wave, and Rossby waves on an equatorial dispersion diagram.]
2. **Mountain Waves** We want to solve the problem of the steady vertical velocity \( w \) induced by a weak mean zonal flow \( U \) of air over a range of periodic, meridionally oriented mountains \( h = h_o \cos(kx) \). A sketch of the problem is below.

Assume the stratification is constant \( (N = N_o) \). We found that assuming the Coriolis parameter was a constant \( (f = f_o) \), the vertical velocity satisfied the following relationships in the absence of a mean flow

\[
w = w_o e^{i(kx + mz - \omega t)}; \quad \omega^2 = (f_o^2 m^2 + N_o^2 k^2)/(k^2 + m^2) \tag{2.1}
\]

(note: remember the current problem is a lot like the problem of a mountain range moving at speed \(-U\) under calm air).

Perhaps with the help of (2.1), calculate the steady vertical velocity induced by \( U \).

**Sketch of flow over a mountain range** \( h = h_o \cos(kx) \)

Let’s suppose the wind is calm, but the mountains are moving westward as:

\( h = h_o \cos(k(x + Ut)) \). Then the vertical velocity at \( z=0 \) is given by:

\( w(z = 0) = \partial h / \partial t = -h_o U k \sin(k(x + Ut)) \). Thus, the complete solution for \( w \) is:

\[
w = -h_o k U \sin(kx + kUt + mz) \text{ where } m^2 = k^2 \left[ \frac{N_o^2 - k^2 U^2}{k^2 U^2 - f_o^2} \right]
\]
If the term in brackets is negative we get exponential decay with height. We can get the solution to the mountain wave problem by shifting this solution zonally at speed $U$. Then we have: $w = -h_0 k U \sin(kx + mz)$ with $m$ given by the expression above.