1) Consider two one dimensional waves whose sea level expression is given by
\[ \eta_1 = \text{real}(A(\exp(ik_1x - \omega_1t))) \quad \text{and} \quad \eta_2 = \text{real}(A(\exp(ik_2x - \omega_2t))). \]
Assume that the differences in frequency and wavenumber are very small (i.e. \( |\omega_1 - \omega_2| \ll |\omega_1| \)).

A) Show that the crests and troughs of either wave move at speed \( \frac{\omega}{k} \) while energy (for example potential energy, which is proportional to \( \eta^2 \)) is transported at speed \( \frac{\partial \omega}{\partial k} \). B) Under what conditions are these two speeds the same?

2) Shallow water gravity waves satisfy the following equation for pressure:
\[ \frac{\partial^2 p}{\partial t^2} - gH \frac{\partial^2 p}{\partial x^2} = 0 \]
Suppose at time \( t=0 \) the pressure has an initial disturbance of the form \( p(-1<x<1)=1 \), but everywhere else, \( p(x)=0 \). Solve for the evolution of the pressure field.

3) (H#7.7) If the surface height perturbation in a shallow water gravity wave is given by \( \eta = \text{Re}(Ae^{j(kx-\omega t)}) \) find the corresponding velocity perturbation. Sketch the phase relationship for an eastward propagating wave.