Meto611 Problem Set #6 Forced and boundary motion

1) a.) Use the shallow water equations to find the equation for shallow water gravity waves* compute the eigenfrequencies $\omega$ and eigenfunctions $\eta_o(x, y)$ of a rectangular basin of constant depth $H$, length $a$, and width $b$, in the absence of rotation.

b.) Suppose the basin was forced by a periodic zonal wind of the form $\tau^* = \tau \sin(\pi x / a) \cos(\pi y / b) \sin(\alpha t)$. The equation now becomes:

$$\frac{\partial^2 \eta}{\partial t^2} - gH \left( \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right) = \frac{\tau \pi}{\rho a} \cos(\pi x / a) \cos(\pi y / b) \sin(\alpha t) \tag{1.1}$$

Try and solve (1.1) subject to the same boundary conditions as before. What happens if $\alpha$ happens to be the same as one of the eigenfrequencies $\omega$?

2) Prove that coastal Kelvin Waves propagate with the coast on their right in the Northern hemisphere and their left in the Southern Hemisphere.

3) a) The North Atlantic has an average depth of 4km. How long does it take a barotropic Kelvin Wave to propagate around it (you need to look at a map to compute the circumference of the North Atlantic Ocean. Assume the equator is a wall)?

b) Because of the rotation rate of the earth the dominant signal in gravity field has a period of 12.42 hours (lunar semidiurnal tide). Is the frequency of tidal forcing close to Kelvin eigenfrequency for the North Atlantic? What effect will this have on the amplitude of the tidal response?

---

*Hint: equation: $\frac{\partial^2 \eta}{\partial t^2} - gH \left( \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right) = 0$ and boundary conditions of no normal flow $(\frac{\partial \eta}{\partial y})_{y=0} = \frac{\partial \eta}{\partial y} = 0$ and $(\frac{\partial \eta}{\partial x})_{x=0} = \frac{\partial \eta}{\partial x} = 0$). Look for periodic solutions to the equation of the form: $\eta = \eta_o(x, y)e^{-i\omega t}$. So $\eta_o(x, y)$ must satisfy

$$-\omega^2 \eta_o - gH \left( \frac{\partial^2 \eta_o}{\partial x^2} + \frac{\partial^2 \eta_o}{\partial y^2} \right) = 0.$$ Look for solutions that vary sinusoidally in the x- and y-directions. What wavelengths will satisfy the equations and boundary conditions?