METO 621

Lesson 7

Blackbody

- Consider a tiny opening in a hollow sphere
- The chance of incoming radiation being reflected back out of the hole is extremely small
- Hence the opening is perfectly absorbing - it is ‘black.
- Within the sphere the radiation will have reached thermal equilibrium.
- The radiation field in the cavity is:
  - Isotropic, homogeneous, and non-polarized
  - Independent of the nature and shape of the cavity walls
  - Dependent only on temperature *T*

Blackbody radiation

- The radiation emanating from the internal surface is called ‘blackbody radiation’
- Radiation in equilibrium with matter, as specified by the Planck function that describes the distribution of the energies of photons in equilibrium with matter as a function of wavelength.
- No strict blackbodies exist, but some objects are approximately black over a limited range of frequencies and/or directions

Max Planck

- **Max Karl Planck** (April 23, 1858 – October 4, 1947) was a German physicist, the founder of the quantum theory, for which he received the Nobel Prize in Physics in 1918.[1]

Planck's spectral distribution law

- Planck introduced in 1901 his hypothesis of quantized oscillators in a radiating body.
- He derived an expression for the hemispherical blackbody spectral radiative flux

$$F_\nu^{BB} = \frac{m_r^2}{c^2}\frac{2\pi h\nu^3}{[\exp(h\nu/k_B T)-1]}$$

Where h is Planck's constant, m_r is the real index of refraction, k_B is Boltzmann's constant

Three forms of the Planck function

$$B_\nu(T) = \frac{2h\nu^3}{c^2\left(e^{\frac{h\nu}{k_B T}} - 1\right)}$$

[Wm^{-2} Hz^{-1} sr^{-1}]

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda k_B T}} - 1\right)} = \frac{c_1}{\lambda^5\left(e^{\frac{c_2}{\lambda T}} - 1\right)}$$

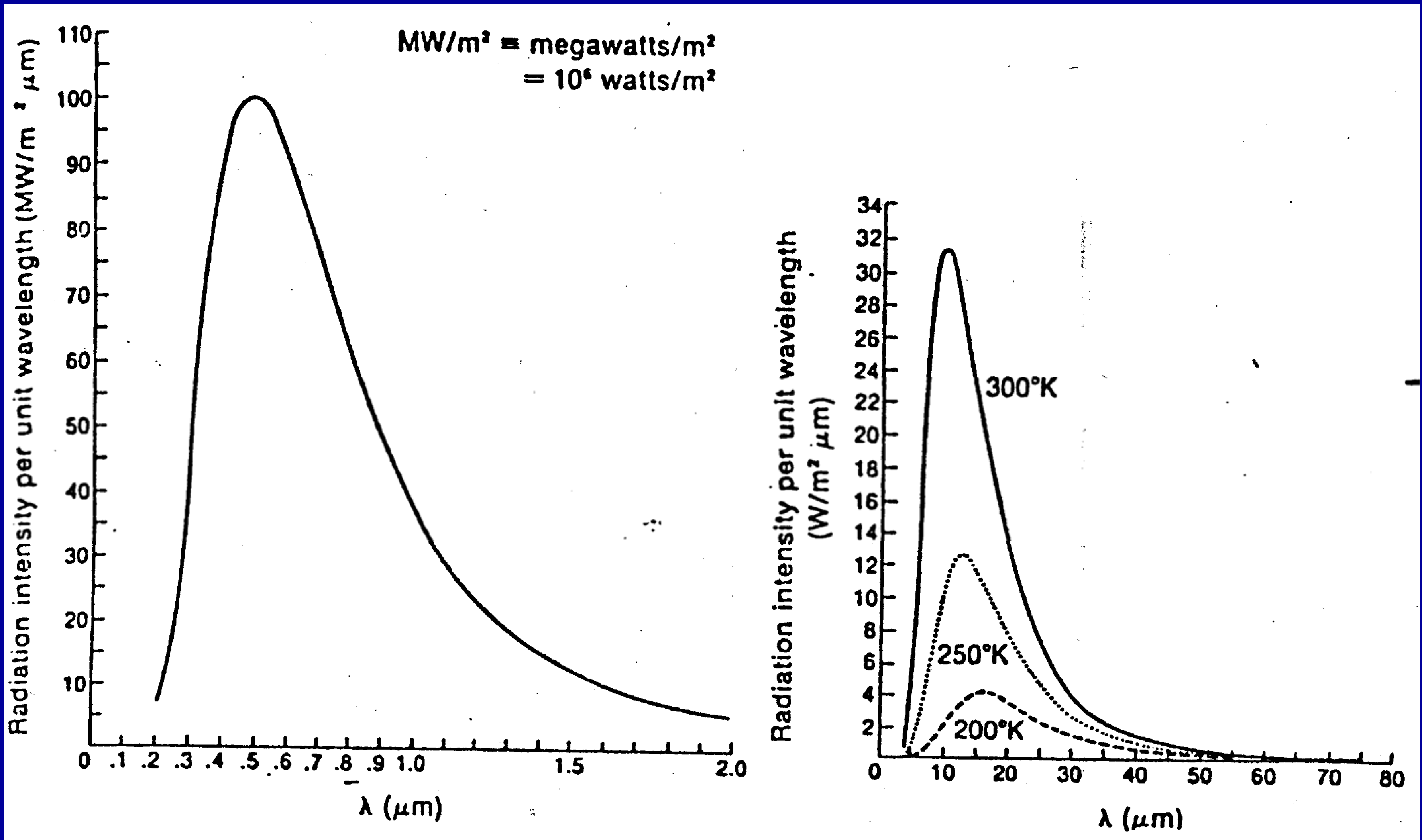
[Wm^{-2} μm^{-1} sr^{-1}]

$$B_{\tilde{\nu}}(T) = \frac{2hc^2\tilde{\nu}^3}{e^{\frac{hc\tilde{\nu}}{k_B T}} - 1}$$

[Wm^{-2} (cm^{-1})$^{-1}$ sr^{-1}]

Units of intensity!

Comparison of solar and earth's blackbody intensity



Planck's spectral distribution law

Approximations for the spectral distribution law are

Wien's limit for high energies:

$$F_\nu^{BB} \approx \frac{m_r^2}{c^2} 2\pi h \nu^3 \exp(-h\nu / k_B T)$$

Rayleigh - Jean's limit for low energies

$$F_\nu^{BB} \approx \frac{2\pi \nu^2 m_r^2 k_B T}{c}$$

Planck's function

As the blackbody radiation is isotropic, the intensity is related to the hemispherical flux through

$$F_\nu^{BB} = \pi I_\nu^{BB} = \pi B_\nu$$

B_ν is known as the Planck function, and has the same units as intensity

$$B_\nu(T) \cong \frac{m_r^2}{c^2} \frac{2h\nu^3}{\left[\exp(h\nu/k_B T) - 1\right]}$$

When dealing with gases m_r is set equal to one

Transformation of Planck function

(1) By differentiating the Planck function and equating the result to zero we find the wavelength corresponding to the maximum:

$$\lambda_m T = 2{,}897.8$$

Wien's displacement law

(2) By integrating the blackbody flux over all frequencies we get :

$$F^{BB} = \sigma_B T^4$$

Stefan-Boltzmann Law

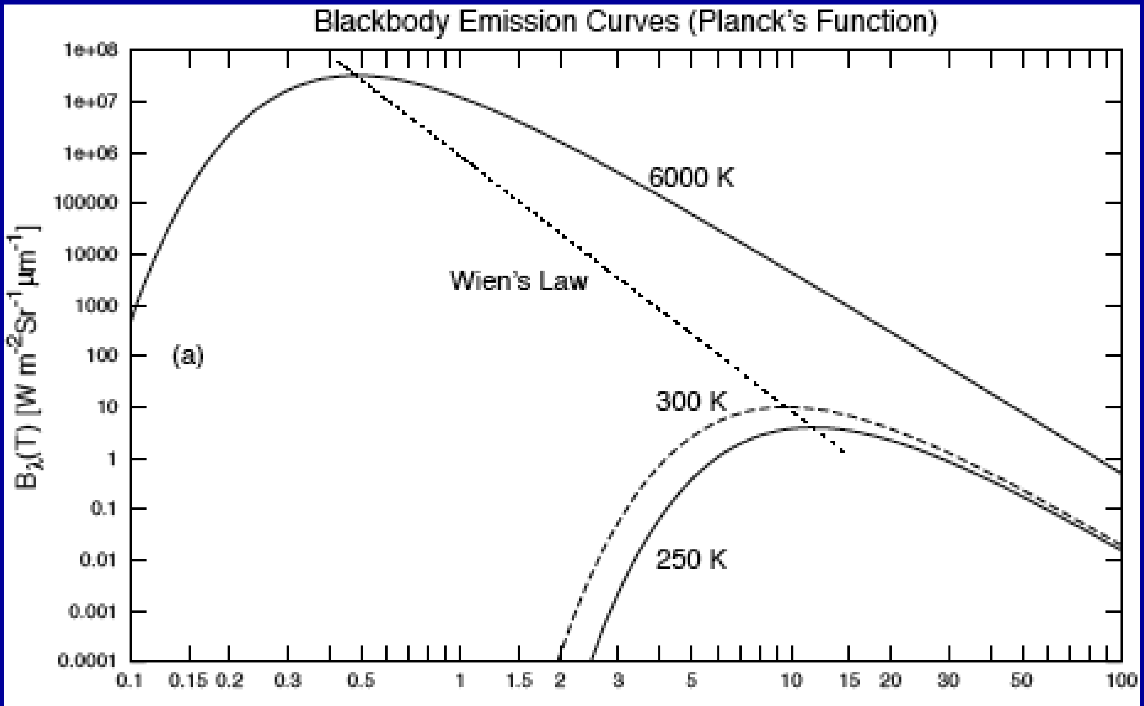
Two important BB laws

Wien's law: Wavelength (frequency, etc.) of maximum emission:

$\lambda_{max}(\mu m) \approx 2897.8/T$

Location of maximun depends on representation (see solved problem at end of notes). Equal wavelength intervals do not correspond to equal frequency intervals:

$$\lambda_2 - \lambda_1 = c\left(\frac{\nu_1 - \nu_2}{\nu_1 \nu_2}\right)$$



Thermal emission from a surface

- Let $I^{+}_{\nu c}(\hat{\Omega})\cos\theta\, d\omega$ be the emitted

energy from a flat surface of temperature T_s, within the solid angle $d\omega$ in the direction Ω. A blackbody would emit $B_\nu(T_s)\cos\theta d\omega$. The spectral directional emittance is defined as

$$\varepsilon(\nu,\hat{\Omega},T_S) = \frac{I^{+}_{\nu e}(\hat{\Omega})\cos\theta d\omega}{B_\nu(T_S)\cos\theta d\omega} = \frac{I^{+}_{\nu e}(\hat{\Omega})}{B_\nu(T_S)}$$

Thermal emission from a surface

- In general ε depends on the direction of emission, the surface temperature, and the frequency of the radiation. A surface for which ε is unity for all directions and frequencies is a blackbody. A hypothetical surface for which ε = constant<1 for all frequencies is a *graybody*.

Flux emittance

- The energy emitted into 2π steradians relative to a blackbody is defined as the flux or bulk emittance

$$\varepsilon(\nu, 2\pi, T_S) \equiv \frac{\int_+ d\omega \cos\theta I_{\nu e}^+(\hat{\Omega})}{\int_+ d\omega \cos\theta B_\nu(T_S)} = \frac{\int_+ d\omega \cos\theta \varepsilon(\nu, \hat{\Omega}, T_S) B_\nu(T_S)}{\pi B_\nu(T_S)}$$

$$= \frac{1}{\pi} \int_+ d\omega \cos\theta \varepsilon(\nu, \hat{\Omega}, T_S)$$

Absorption by a surface

- Let a surface be illuminated by a downward intensity I. Then a certain amount of this energy will be absorbed by the surface. We define the spectral directional absorptance as:

$$\alpha(\nu, -\hat{\Omega}', T_S) = \frac{I_{\nu a}^{-}(\hat{\Omega}')\cos\theta' d\omega'}{I_{\nu}^{-}(\hat{\Omega}')\cos\theta' d\omega'} = \frac{I_{\nu a}^{-}(\hat{\Omega}')}{I_{\nu}^{-}(\hat{\Omega}')}$$

- The minus sign emphasizes the downward direction of the incident radiation

Absorption by a surface

- Similar to emission, we can define a flux absorptance

$$\alpha(\nu,-2\pi,T_S) = \frac{\int_{-1} d\omega' \cos\theta' \, \alpha(\nu,-\hat{\Omega}',T_S) I_\nu^-(\hat{\Omega}')}{F_\nu^-}$$

Kirchoff Law

- Kirchoff showed that for an opaque surface

$$\alpha(\nu, -\hat{\Omega}, T_S) = \varepsilon(\nu, \hat{\Omega}, T_S)$$

- That is, a good absorber is also a good emitter, and vice-versa

- The law is theoretically valid only within an isothermal enclosure in thermodynamic equilibrium, but in practice it is broadly acceptable.

- But generally does not apply to angularly and spectrally integrated quantities.

Absorption and Scattering in Planetary Media

- Kirchoff's Law for volume absorption and Emission

$$\varepsilon_{\nu}(\nu,T)=\frac{\alpha(\nu,T)}{k(\nu)}$$

The volume emittance is proportional to the absorption coefficient

Surface reflection : the BRDF

Consider a downward beam with intensity $I_\nu^-(\hat{\Omega})$.

The energy incident on a flat surface is $I_\nu^-(\hat{\Omega})\cos\theta\, d\omega'$.

Let the intensity of the reflected light around the direction $\hat{\Omega}$ within a solid angle $d\omega$ be $dI_{\nu r}^-$ then

$$\rho(\nu,-\hat{\Omega}',\hat{\Omega}) = \frac{dI_{\nu r}^-(\hat{\Omega})}{I_\nu(\hat{\Omega}')\cos\theta'\, d\omega'}$$

where $\rho(\nu,-\hat{\Omega}',\hat{\Omega})$ is the bidirectional relfectance distribution function, or BRDF.

BRDF

The total reflected intensity in the direction $\hat{\Omega}$, from all beams is

$$I_{\nu r}^{+}(\hat{\Omega}) = \int_{-} dI_{\nu r}^{+}(\hat{\Omega}) = \int_{-} d\omega' \cos\theta' \rho(\nu, -\hat{\Omega}', \hat{\Omega}) I_{\nu}^{-}(\hat{\Omega}')$$

If a reflecting surface has a BRDF which is independent of both the incidence and observation directions, then it is called a Lambert surface.

In this case $\rho(\nu, -\hat{\Omega}', \hat{\Omega}) = \rho_L(\nu)$, and

$$I_{\nu r}^{+} = \rho_L(\nu) \int_{-} d\omega' \cos\theta' I_{\nu}^{-}(\hat{\Omega}') = \rho_L(\nu) F$$

Surface reflectance - BRDF

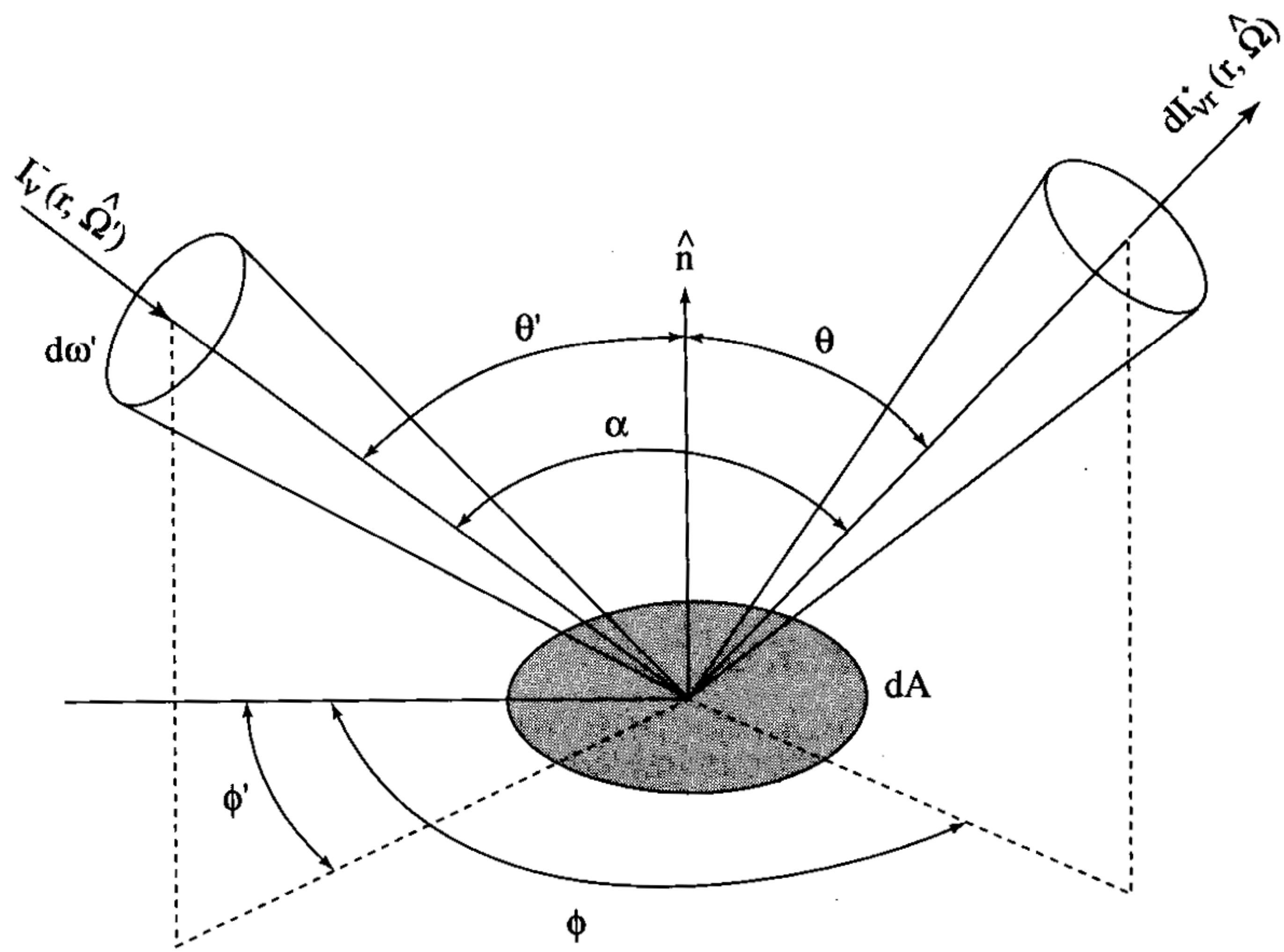
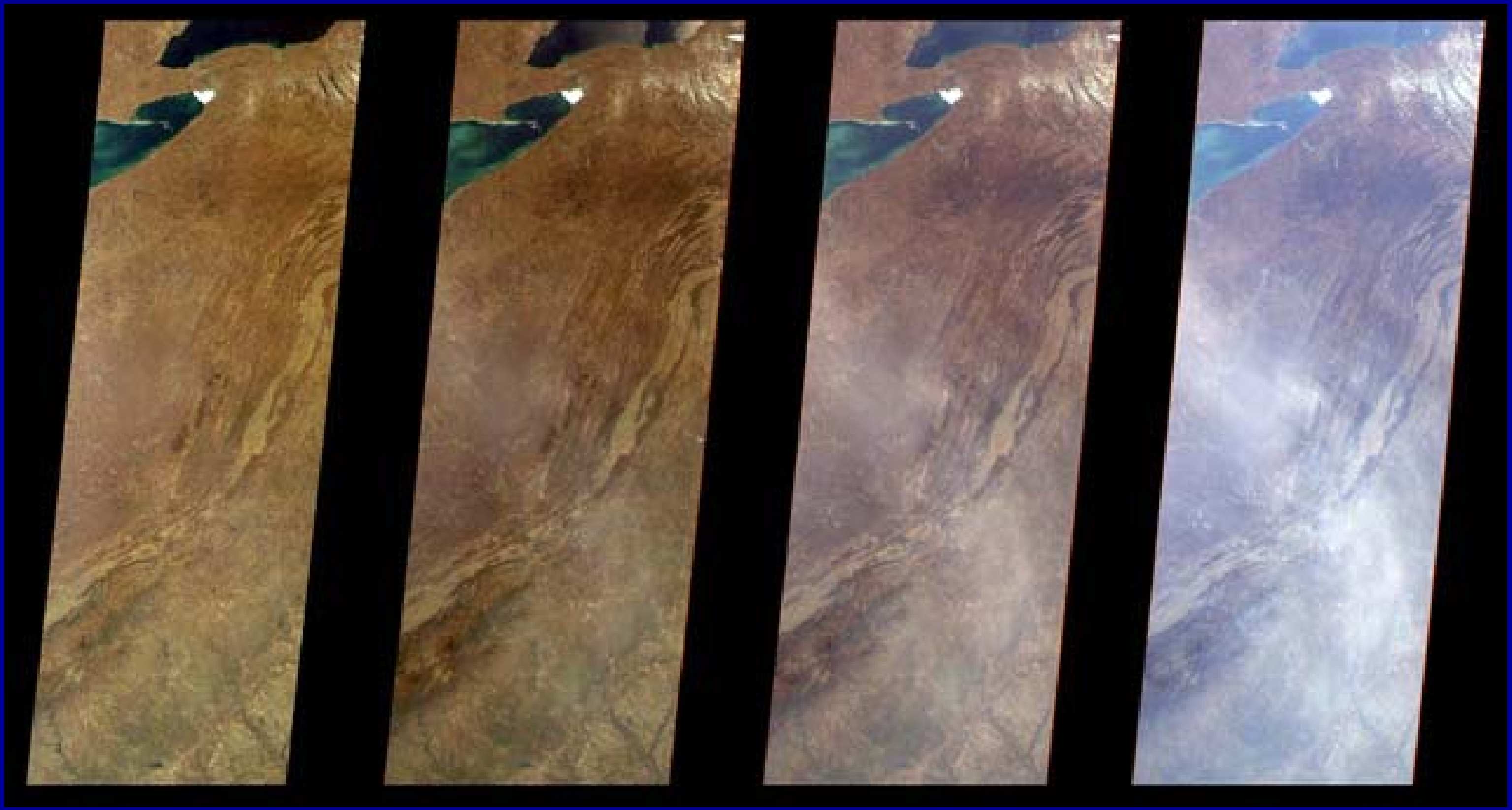


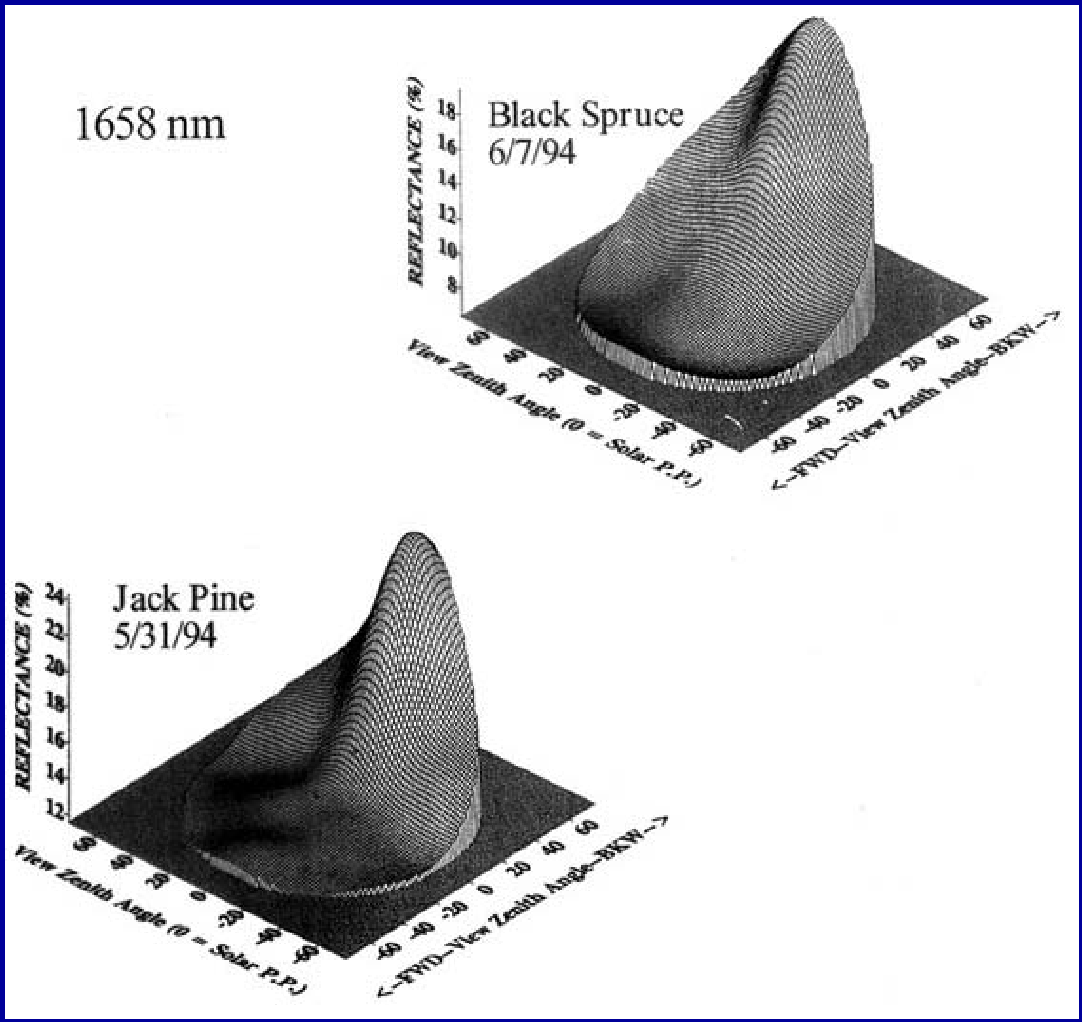
Figure 5.1 Geometry and symbols for the definition of the BRDF. The angle α is the backscattering angle.

BRDF conveys info of the target as seen from this MISR Image of Haze

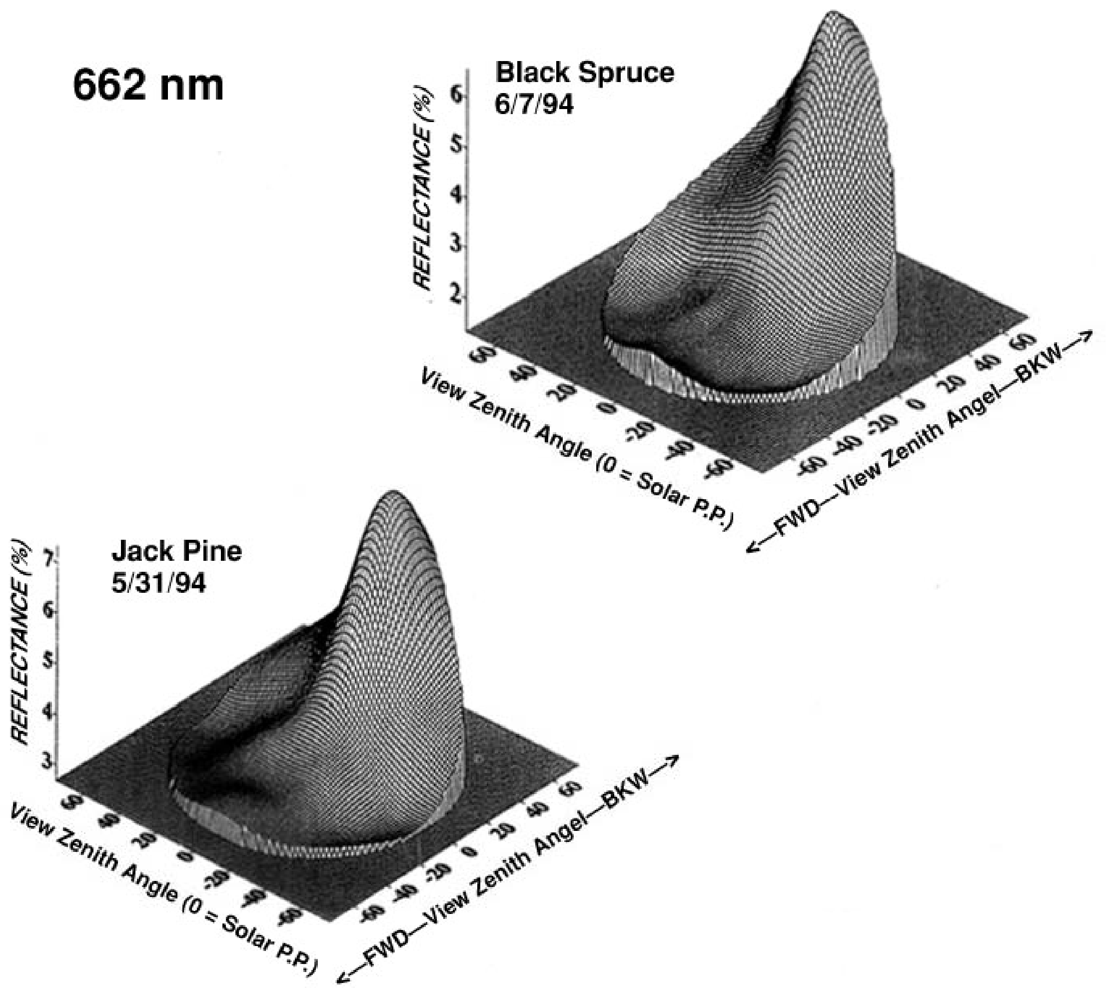


- In this MISR view spanning from Lake Ontario to Georgia, the increasingly oblique view angles reveal a pall of haze over the Appalachian Mountains

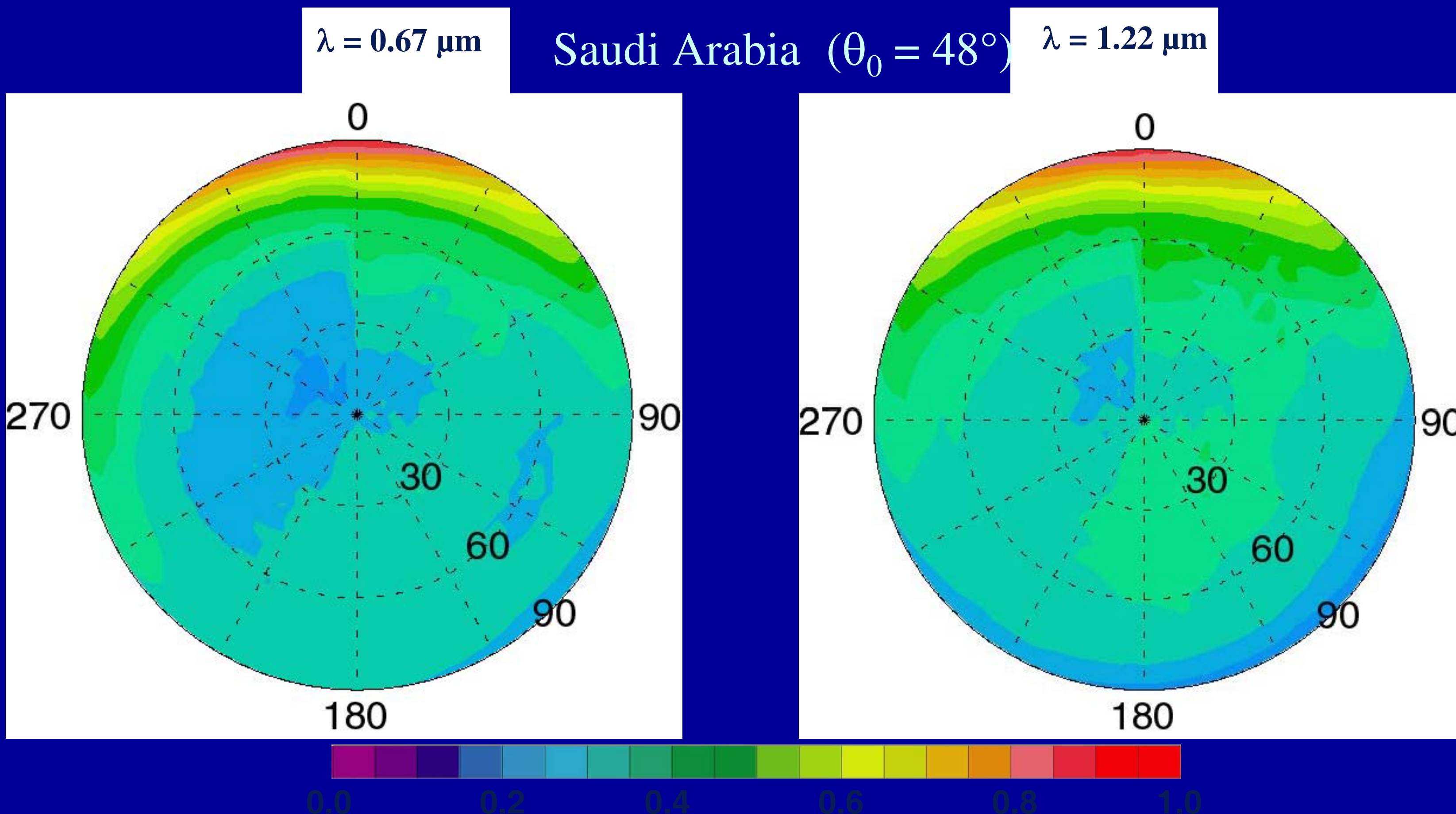
Bidirectional Reflectance of Black Spruce & Jack Pine



Bidirectional Reflectance of Black Spruce & Jack Pine



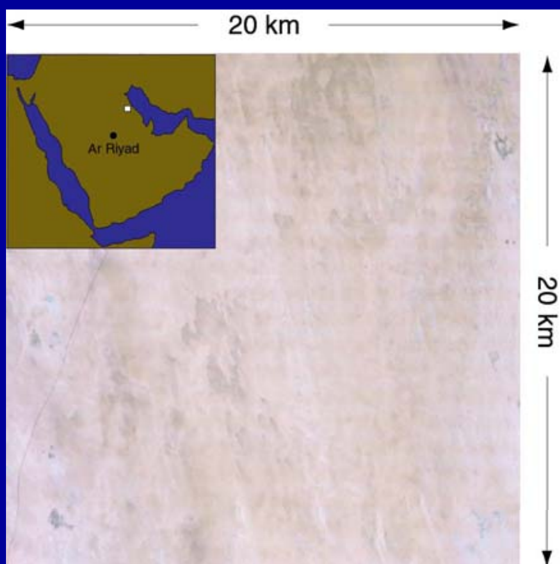
Bidirectional Reflectance - Desert

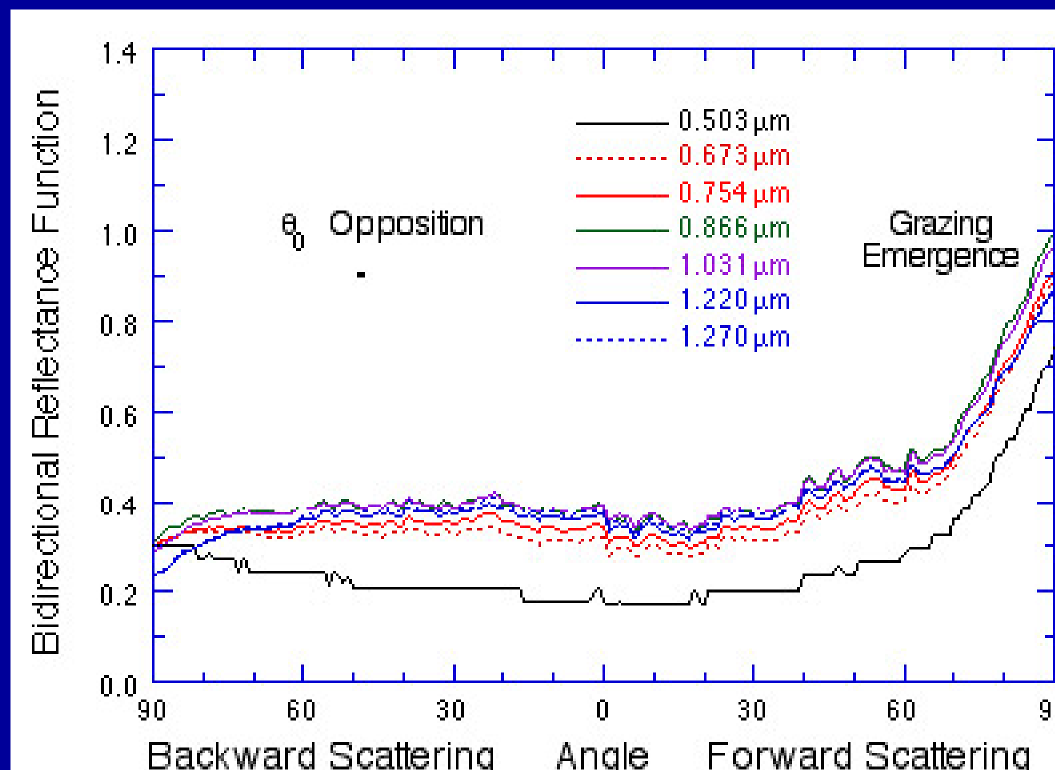


Bidirectional Reflectance - Desert

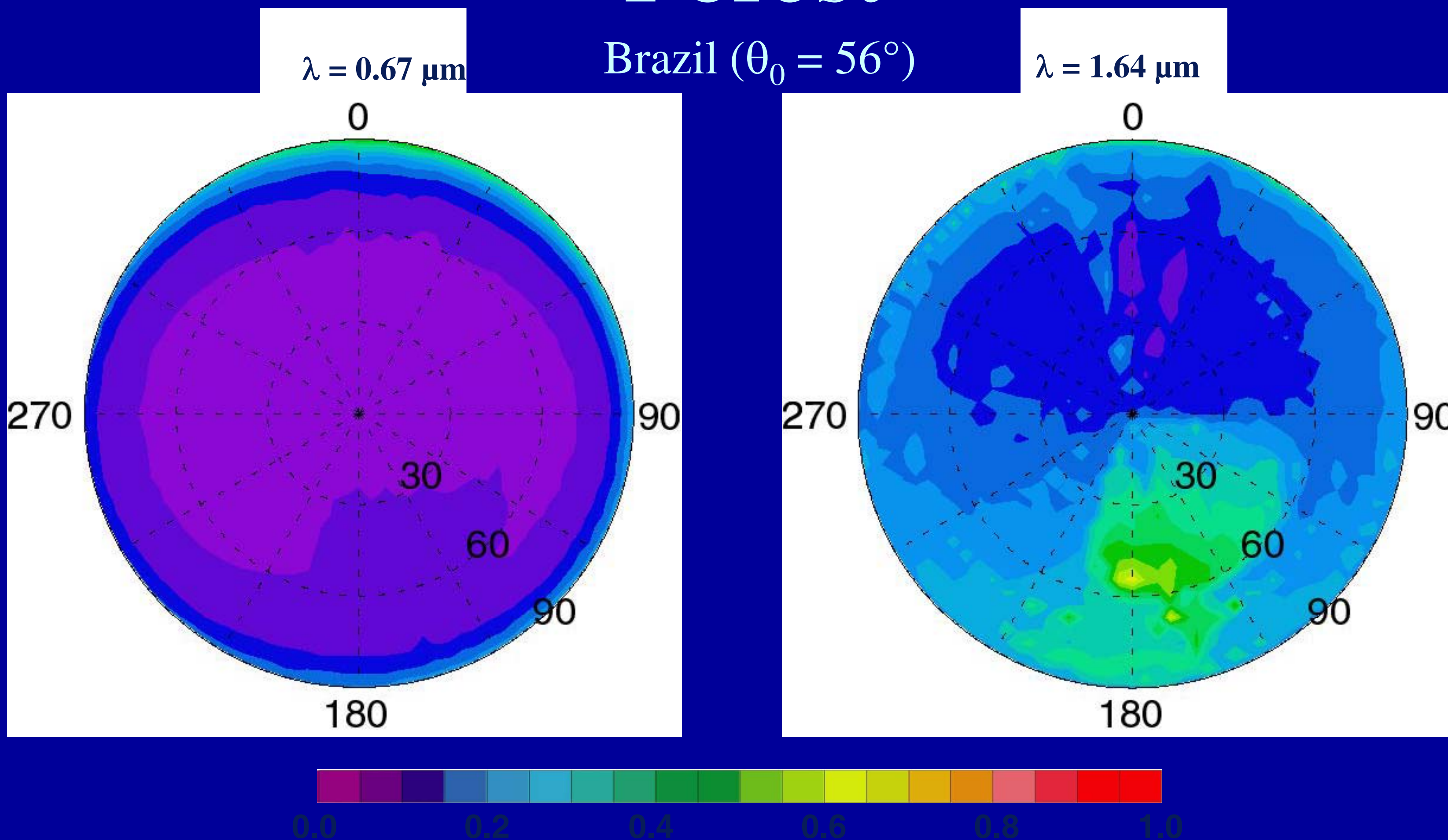
Saudi Arabia ($\theta_0 = 48°$)

Saudi Arabian Desert
(Landsat Image)



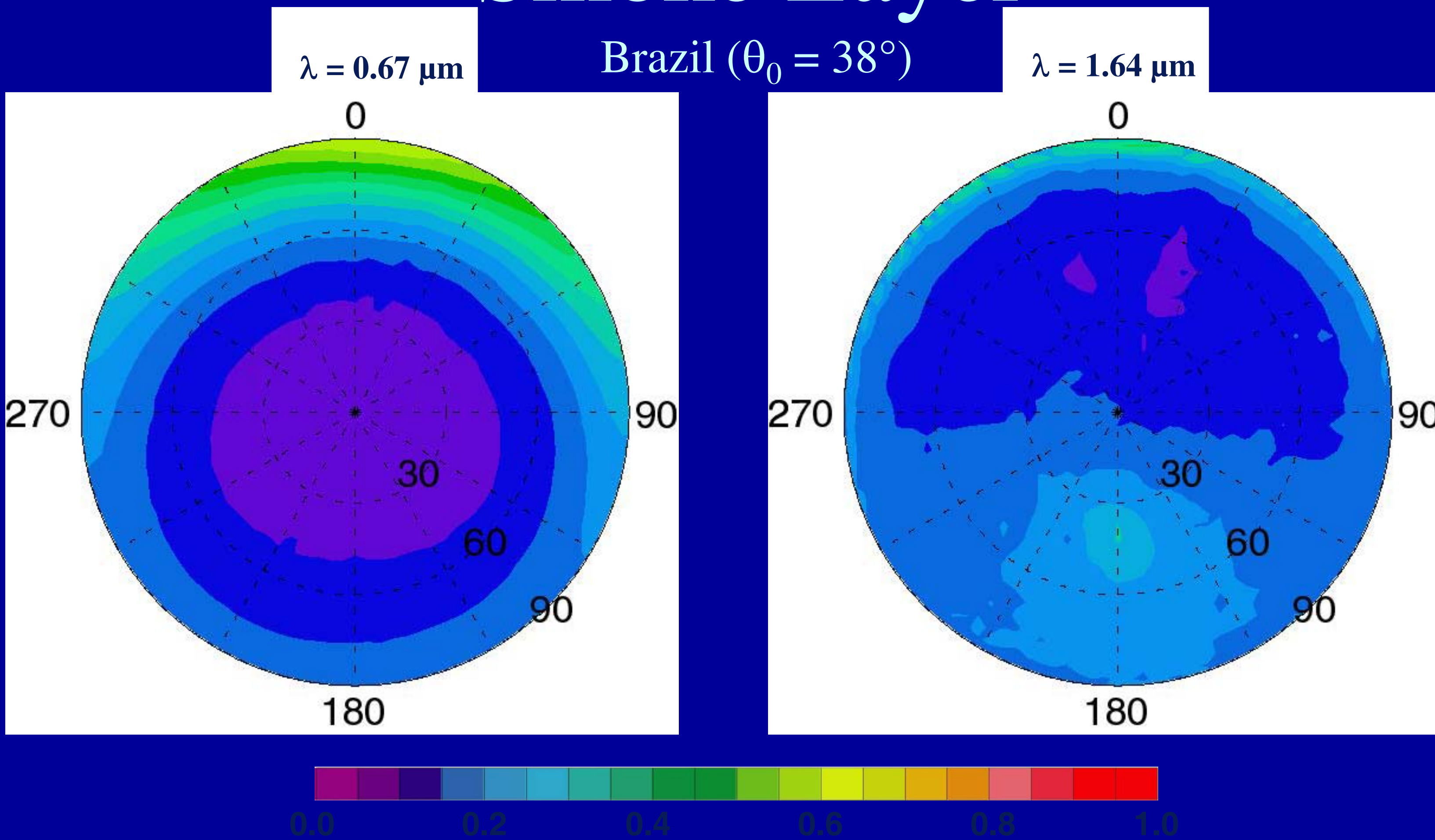


Bidirectional Reflectance - Dense Forest



Bidirectional Reflectance - Smoke Layer

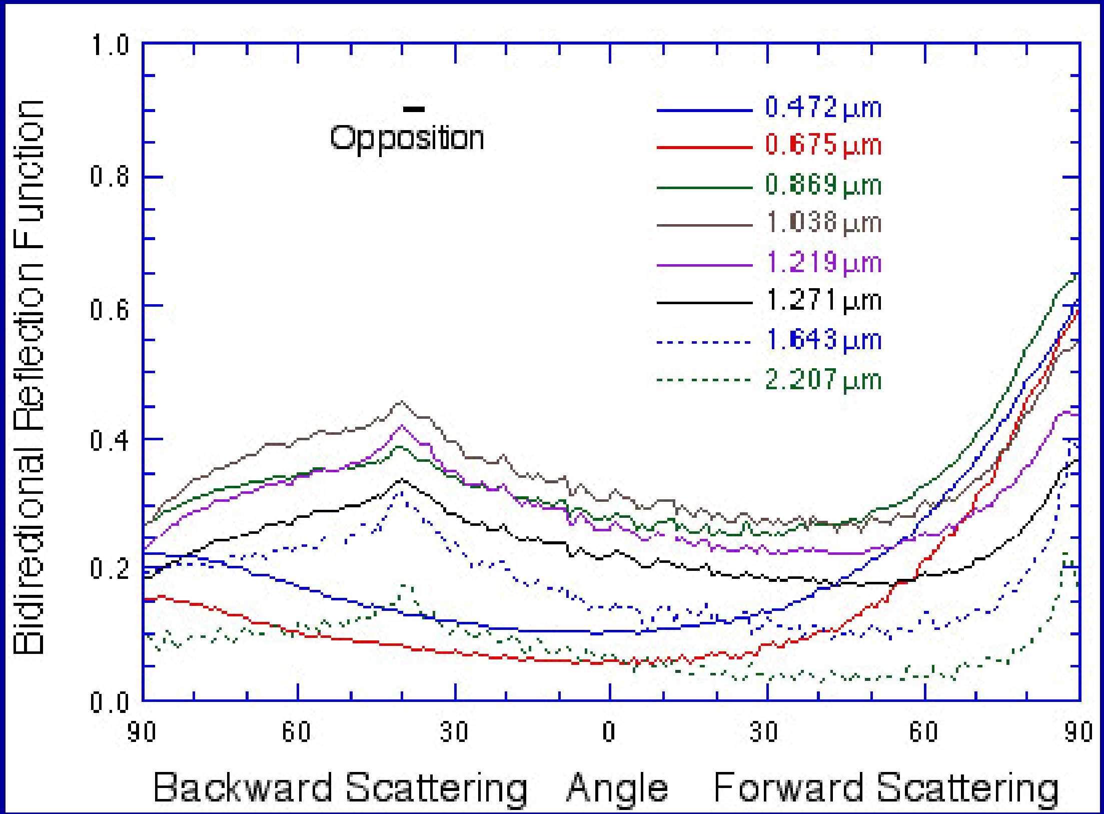
Brazil (θ_0 = 38°)



Bidirectional Reflectance - Smoke Layer

Brazil ($\theta_0 = 38°$)

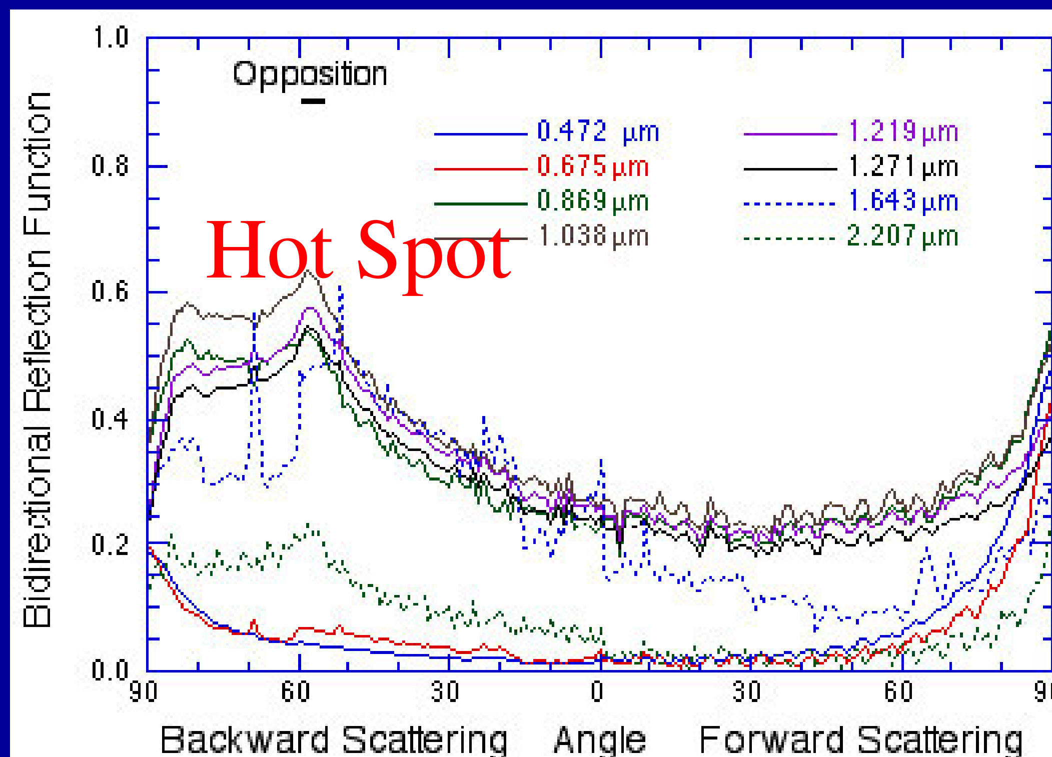
Smoke Layer



Bidirectional Reflectance - Dense Forest

Brazil ($\theta_0 = 56°$)

Dense Forest



Homework 2, Due Feb 28

- See the WORD file attached.