Blackbody

• Consider a tiny opening in a hollow sphere
• The chance of incoming radiation being reflected back out of the hole is extremely small
• Hence the opening is perfectly absorbing - it is ‘black.
• Within the sphere the radiation will have reached thermal equilibrium.
• The radiation field in the cavity is:
  – Isotropic, homogeneous, and non-polarized
  – Independent of the nature and shape of the cavity walls
  – Dependent only on temperature $T$
Blackbody radiation

• The radiation emanating from the internal surface is called ‘blackbody radiation’

• Radiation in equilibrium with matter, as specified by the Planck function that describes the distribution of the energies of photons in equilibrium with matter as a function of wavelength.

• No strict blackbodies exist, but some objects are approximately black over a limited range of frequencies and/or directions
Max Planck

- Max Karl Planck (April 23, 1858 – October 4, 1947) was a German physicist, the founder of the quantum theory, for which he received the Nobel Prize in Physics in 1918. [1]
Planck’s spectral distribution law

- Planck introduced in 1901 his hypothesis of quantized oscillators in a radiating body.
- He derived an expression for the hemispherical blackbody spectral radiative flux

\[
F_{\nu}^{BB} = \frac{m_r^2}{c^2} \frac{2 \pi h \nu^3}{\left[\exp\left(\frac{h \nu}{k_B T}\right) - 1\right]}
\]

Where \( h \) is Planck’s constant, \( m_r \) is the real index of refraction, \( k_B \) is Boltzmann’s constant.
Three forms of the Planck function

\[
B_\nu(T) = \frac{2h\nu^3}{c^2 \left( e^{\frac{h\nu}{k_B T}} - 1 \right)}
\]

\[
B_\lambda(T) = \frac{2hc^2}{\lambda^5 \left( e^{\frac{hc}{\lambda k_B T}} - 1 \right)} = \frac{c_1}{\lambda^5 \left( e^{\frac{c_2}{\lambda T}} - 1 \right)}
\]

\[
B_{\varphi_0}(T) = \frac{2hc^2 \varphi_0}{hce^{k_B T} - 1}
\]

Units of intensity:

- \(\text{Wm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}\) for \(B_\nu\)
- \(\text{Wm}^{-2} \mu\text{m}^{-1} \text{ sr}^{-1}\) for \(B_\lambda\)
- \(\text{Wm}^{-2} (\text{cm}^{-1})^{-1} \text{ sr}^{-1}\) for \(B_{\varphi_0}\)
Comparison of solar and earth’s blackbody intensity
Planck’s spectral distribution law

Approximations for the spectral distribution law are

Wien's limit for high energies:

\[ F^B_B \approx \frac{m_r^2}{c^2} 2\pi h \nu^3 \exp\left(-\frac{h \nu}{k_B T}\right) \]

Rayleigh-Jean's limit for low energies

\[ F^B_B \approx \frac{2\pi \nu^2 m_r^2 k_B T}{c} \]
Planck’s function

As the blackbody radiation is isotropic, the intensity is related to the hemispherical flux through

\[ F_{\nu}^{BB} = \pi I_{\nu}^{BB} = \pi B_{\nu} \]

\( B_{\nu} \) is known as the Planck function, and has the same units as intensity

\[ B_{\nu}(T) \approx \frac{m_r^2}{c^2} \frac{2h\nu^3}{\left[\exp(h\nu/k_BT) - 1\right]} \]

When dealing with gases \( m_r \) is set equal to one
Transformation of Planck function

(1) By differentiating the Planck function and equating the result to zero we find the wavelength corresponding to the maximum:

\[ \lambda_m T = 2,897.8 \]

Wien’s displacement law

(2) By integrating the blackbody flux over all frequencies we get:

\[ F_{BB} = \sigma_B T^4 \]

Stefan-Boltzmann Law
Two important BB laws

Wien’s law: Wavelength (frequency, etc.) of maximum emission:

\[ \lambda_{\text{max}}(\mu\text{m}) \approx \frac{2897.8}{T} \]

Location of maximum depends on representation (see solved problem at end of notes). Equal wavelength intervals do not correspond to equal frequency intervals:

\[ \lambda_2 - \lambda_1 = c \left( \frac{\nu_1 - \nu_2}{\nu_1 \nu_2} \right) \]
Thermal emission from a surface

- Let $I_{ve}^+ (\hat{\Omega}) \cos \theta d\omega$ be the emitted energy from a flat surface of temperature $T_s$, within the solid angle $d\omega$ in the direction $\Omega$. A blackbody would emit $B_\nu(T_s) \cos \theta d\omega$. The spectral directional emittance is defined as

$$\varepsilon (\nu, \hat{\Omega}, T_S) = \frac{I_{ve}^+ (\hat{\Omega}) \cos \theta d\omega}{B_\nu (T_S) \cos \theta d\omega} = \frac{I_{ve}^+ (\hat{\Omega})}{B_\nu (T_S)}$$
Thermal emission from a surface

• In general $\varepsilon$ depends on the direction of emission, the surface temperature, and the frequency of the radiation. A surface for which $\varepsilon$ is unity for all directions and frequencies is a blackbody. A hypothetical surface for which $\varepsilon = \text{constant} < 1$ for all frequencies is a graybody.
Flux emittance

- The energy emitted into $2\pi$ steradians relative to a blackbody is defined as the flux or bulk emittance

$$
\varepsilon(\nu, 2\pi, T_s) \equiv \frac{\int \omega \cos \theta I_{\nu e}^+(\hat{\Omega})}{\int \omega \cos \theta B_\nu(T_s)} = \frac{\int \omega \cos \theta \varepsilon(\nu, \hat{\Omega}, T_s) B_\nu(T_s)}{\pi B_\nu(T_s)}
$$

$$
= \frac{1}{\pi} \int \omega \cos \theta \varepsilon(\nu, \hat{\Omega}, T_s)
$$
Absorption by a surface

• Let a surface be illuminated by a downward intensity $I$. Then a certain amount of this energy will be absorbed by the surface. We define the spectral directional absorptance as:

$$\alpha(\nu,-\hat{\Omega}', T_s) = \frac{I_{va}(\hat{\Omega}') \cos \theta' d\omega'}{I_{v}(\hat{\Omega}') \cos \theta' d\omega'} = \frac{I_{va}(\hat{\Omega}')}{I_{v}(\hat{\Omega}')}$$

• The minus sign emphasizes the downward direction of the incident radiation.
Absorption by a surface

- Similar to emission, we can define a flux absorptance

\[
\alpha(v, -2\pi, T_S) = \frac{-1}{F_v^-} \int d\omega' \cos \theta' \alpha(v, -\hat{\Omega}', T_S) I_v^- (\hat{\Omega}')
\]
Kirchhoff Law

- Kirchhoff showed that for an opaque surface
  \[ \alpha(\nu, -\hat{\Omega}, T_s) = \varepsilon(\nu, \hat{\Omega}, T_s) \]

- That is, a good absorber is also a good emitter, and vice-versa

- The law is theoretically valid only within an isothermal enclosure in thermodynamic equilibrium, but in practice it is broadly acceptable.

- But generally does not apply to angularly and spectrally integrated quantities.
Absorption and Scattering in Planetary Media

- Kirchoff’s Law for volume absorption and Emission

\[ \varepsilon_{\nu}(\nu, T) = \frac{\alpha(\nu, T)}{k(\nu)} \]

The volume emittance is proportional to the absorption coefficient.
Surface reflection: the BRDF

Consider a downward beam with intensity $I_v^{-}(\hat{\Omega})$.
The energy incident on a flat surface is $I_v^{-}(\hat{\Omega})\cos \theta d\omega'$.
Let the intensity of the reflected light around the direction $\hat{\Omega}$ within a solid angle $d\omega$ be $dI_{vr}$, then

$$\rho(\nu,-\hat{\Omega}',\hat{\Omega}) = \frac{dI_{vr}(\hat{\Omega})}{I_v(\hat{\Omega}')\cos \theta' d\omega'}$$

where $\rho(\nu,-\hat{\Omega}',\hat{\Omega})$ is the bidirectional reflectance
distribution function, or BRDF.
The total reflected intensity in the direction $\hat{\Omega}$, from all beams is

$$I_{vr}^+(\hat{\Omega}) = \int dI_{vr}^+(\hat{\Omega}) = \int d\omega' \cos \theta' \rho(\nu,-\hat{\Omega}',\hat{\Omega})I_{v}^{-}(\hat{\Omega}')$$

If a reflecting surface has a BRDF which is independent of both the incidence and observation directions, then it is called a Lambert surface.

In this case $\rho(\nu,-\hat{\Omega}',\hat{\Omega}) = \rho_L(\nu)$, and

$$I_{vr}^+ = \rho_L(\nu) \int d\omega' \cos \theta' I_{v}^{-}(\hat{\Omega}') = \rho_L(\nu) F$$
**Figure 5.1** Geometry and symbols for the definition of the BRDF. The angle $\alpha$ is the backscattering angle.
BRDF conveys info of the target as seen from this MISR Image of Haze

- In this MISR view spanning from Lake Ontario to Georgia, the increasingly oblique view angles reveal a pall of haze over the Appalachian Mountains
Bidirectional Reflectance of Black Spruce & Jack Pine
Bidirectional Reflectance of Black Spruce & Jack Pine
Bidirectional Reflectance - Desert

\[ \lambda = 0.67 \, \text{µm} \quad \text{Saudi Arabia} \quad \theta_0 = 48^\circ \quad \lambda = 1.22 \, \text{µm} \]
Bidirectional Reflectance - Desert
Saudi Arabia ($\theta_0 = 48^\circ$)
Bidirectional Reflectance - Dense Forest

Brazil ($\theta_0 = 56^\circ$)

\[ \lambda = 0.67 \text{ µm} \]

\[ \lambda = 1.64 \text{ µm} \]
Bidirectional Reflectance - Smoke Layer

λ = 0.67 µm

Brazil (θ₀ = 38°)

λ = 1.64 µm
Bidirectional Reflectance - Smoke Layer

Brazil ($\theta_0 = 38^\circ$)
Bidirectional Reflectance - Dense Forest

Brazil ($\theta_0 = 56^\circ$)

Dense Forest

Hot Spot
Homework 2, Due Feb 28

• See the WORD file attached.