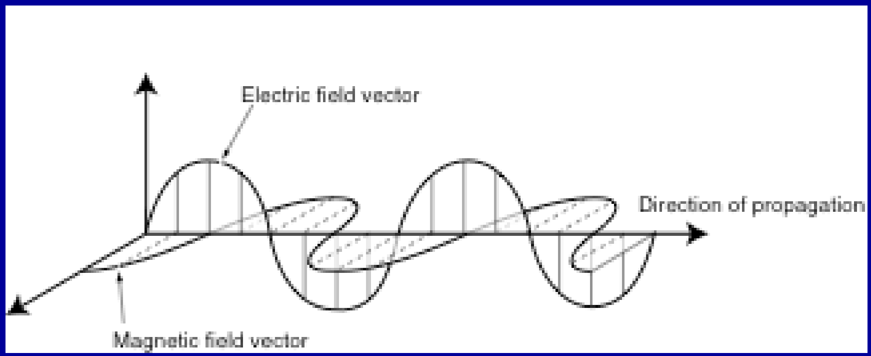
Lesson 3

Definitions of Radiative Quantities



Wavelength (λ): µm (10^{-6} m), nm (10^{-9} m), A (10^{-10} m)

Wavenumber $(\tilde{\nu})$ = 1/λ: m^{-1} (# of waves in unit dist.)

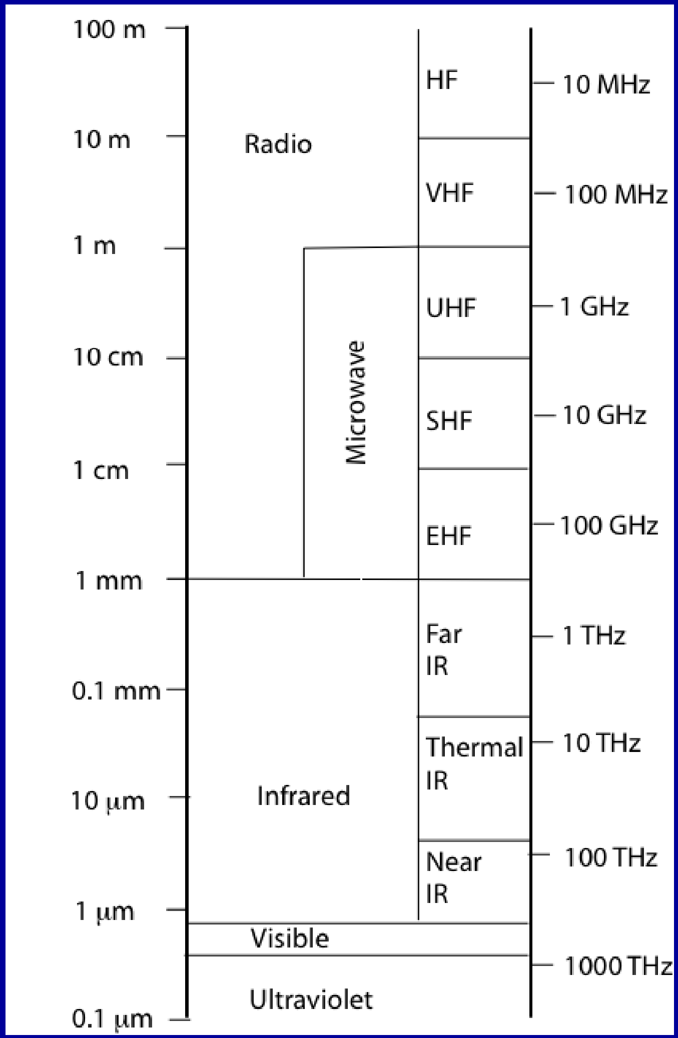
Frequency (ν) = c/λ: s^{-1} = Hz (Hertz) (# of waves passing a point in 1 s)

$c = 3.10^8$ m/s (speed of light)

Amplitude(A) (not used very often)

Energy (E): W ($E \sim A \cdot \nu$)

Wavelength Frequency



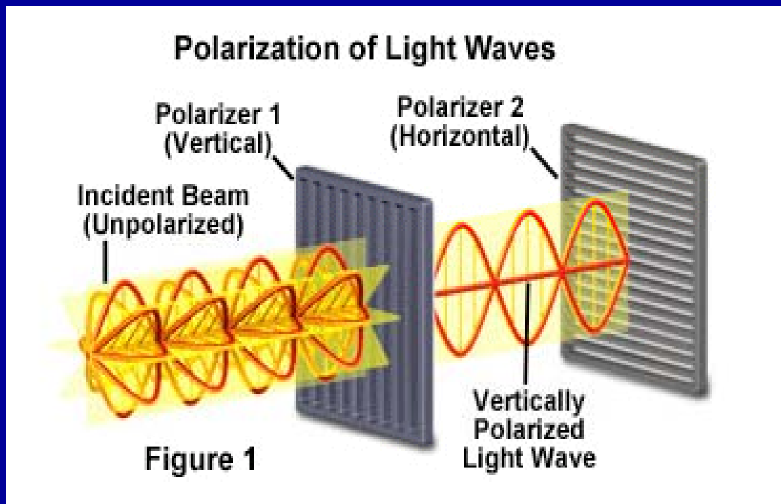
Basic State Variables Describing Radiative Transfer

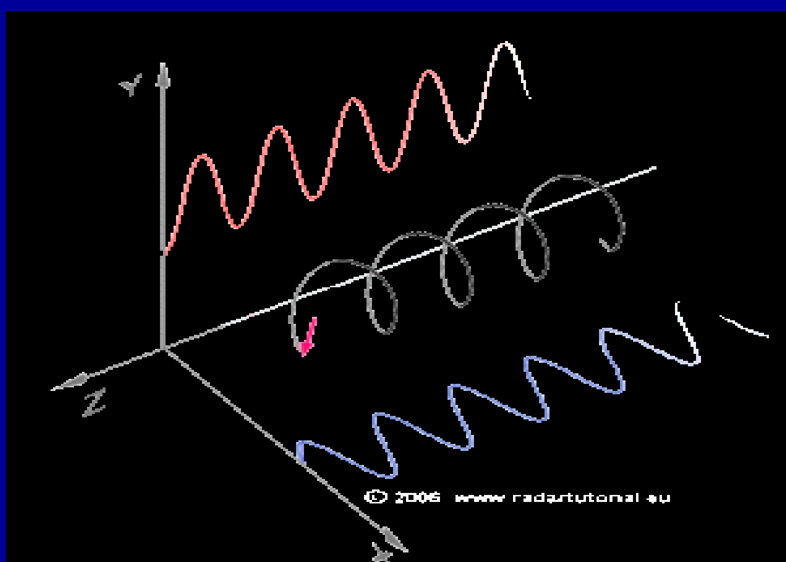
- In this course we are mostly concerned with the flow of radiative energy through the atmosphere.
- Of central importance is the scalar intensity.
- We will refer to polarization effects but not in any detail. Our focus is on the scalar approximation, in contrast with the vector description.
- Its specification as a function of position, direction, and frequency conveys all of the desired information about the radiation field (except for polarization).

Polarization

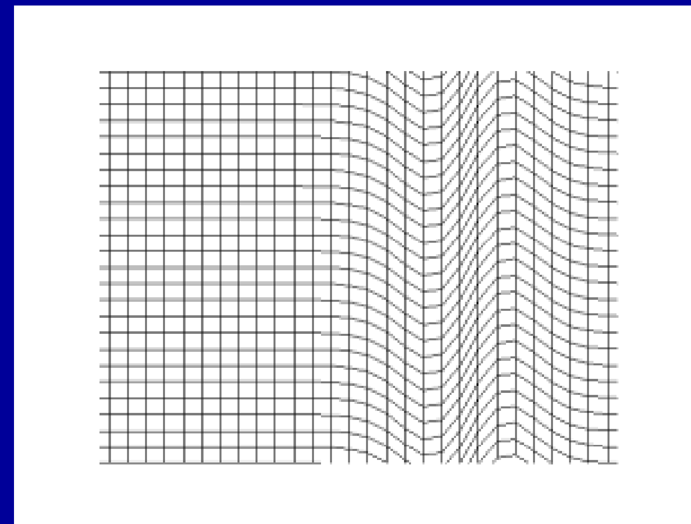
- **Polarization** (also **polarisation**) is a property of certain types of waves that describes the orientation of their oscillations. Electromagnetic waves, such as light, and gravitational waves exhibit polarization; acoustic waves (sound waves) in a gas or liquid do not have polarization because the direction of vibration and direction of propagation are the same.

Polarization
through filtering

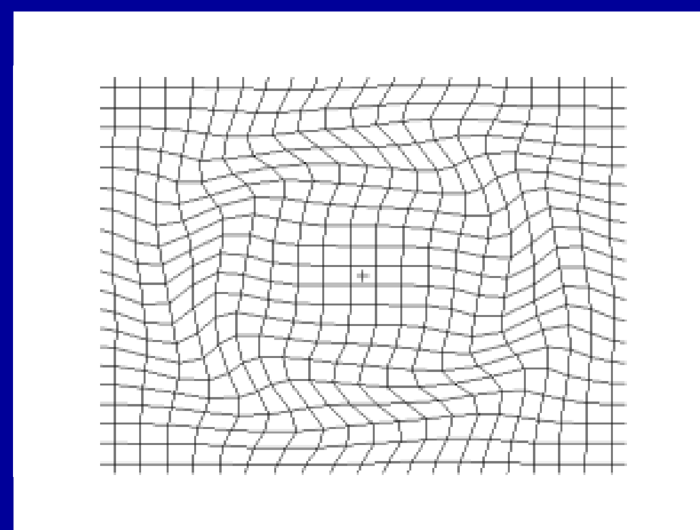




Linearly polarized waves



Circularly polarized wave



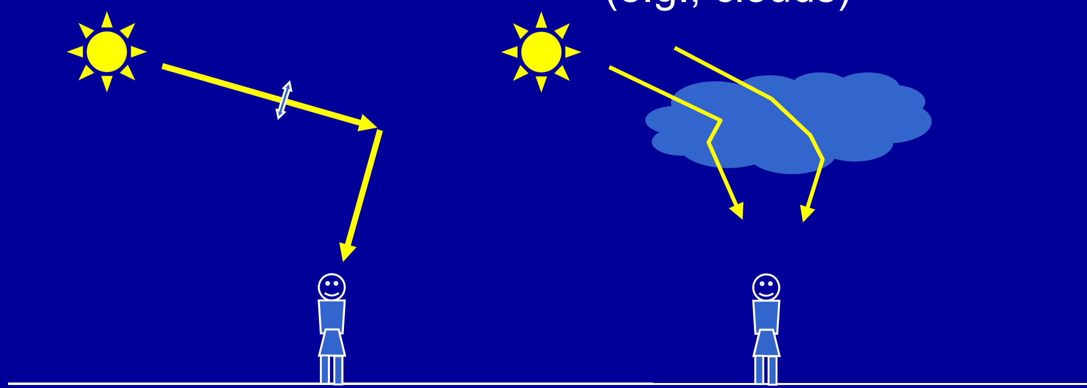
Polarization

- Most sources of electromagnetic radiation contain a large number of atoms or molecules that emit light. The orientation of the electric fields produced by these emitters may not be correlated, in which case the light is said to be *unpolarized*. If there is partial correlation between the emitters, the light is *partially polarized*. One may describe the light in terms of the degree of polarization.
- The **Stokes parameters** are a set of values that describe the polarization state of electromagnetic radiation in terms of its total intensity (*I*), (fractional) degree of polarization (*p*), and the shape parameters of the polarization ellipse.

Polarization in the atmosphere

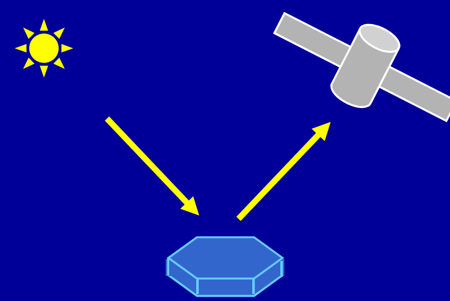
Clear-sky polarization

Multiple scattering reduces polarization (e.g., clouds)

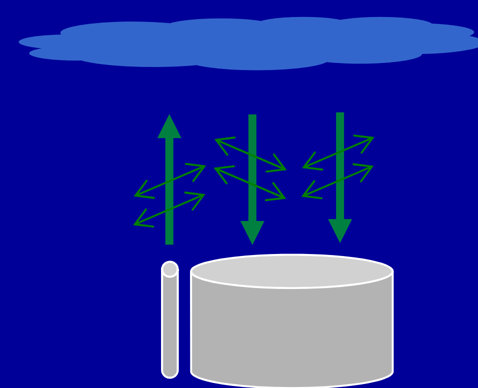


Atmospheric remote sensing

Passive: Ice clouds and aerosol from satellites



Active: Clouds and aerosol using lidar



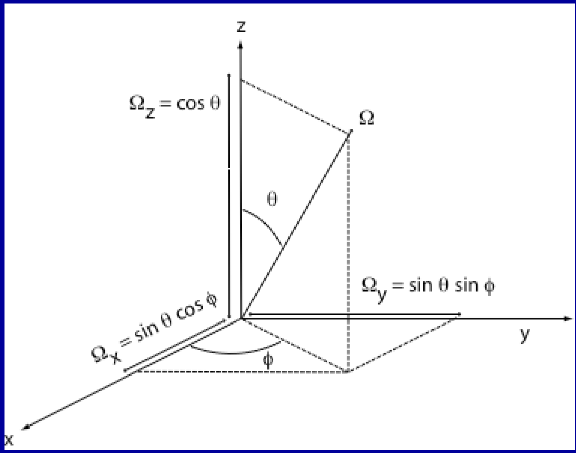
Geometrical optics

- For our purposes it is convenient to use the concept of incoherent (non-interfering) beams of radiation
- A beam is defined in analogy with a plane wave. It carries energy in a specific direction and has infinite extension in the transverse direction.
- When a beam of sunlight is scattered by the Earth's atmosphere it is split into an infinite number of incoherent beams propagating in different directions.
- It is also convenient to define an angular beam as an incoherent sum of beams propagating in various directions inside a small cone of solid angle $d\omega$, centered around a direction Ω.

How we can describe radiation

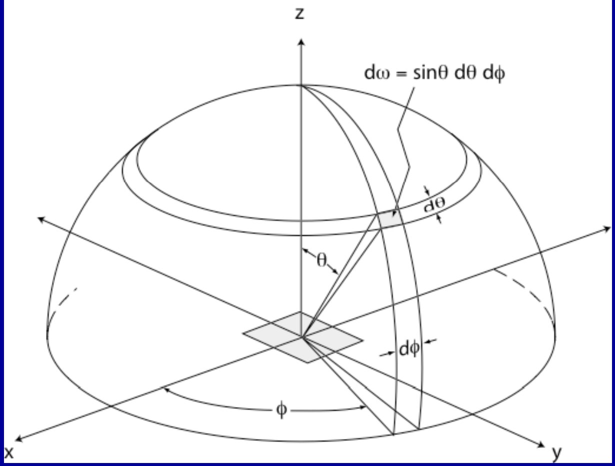
Direction, wiggliness, polarization, radiative quantities (e.g., flux, radiance, albedo)
surface reflection, concept of extinction, radiative transfer equation

Direction



θ = zenith angle
ϕ= azimuth (from North to East)
$u = \cos(\theta)$
$\mu = |u|$
Subscript 0: radiation coming from Sun

If interested in not a single specific direction: solid angle (ω)



$$\omega = \frac{surface}{radius^2}$$

For entire sphere: $\omega = \frac{4\pi r^2}{r^2} = 4\pi$

(steradian, unitless)

Flow of radiative energy

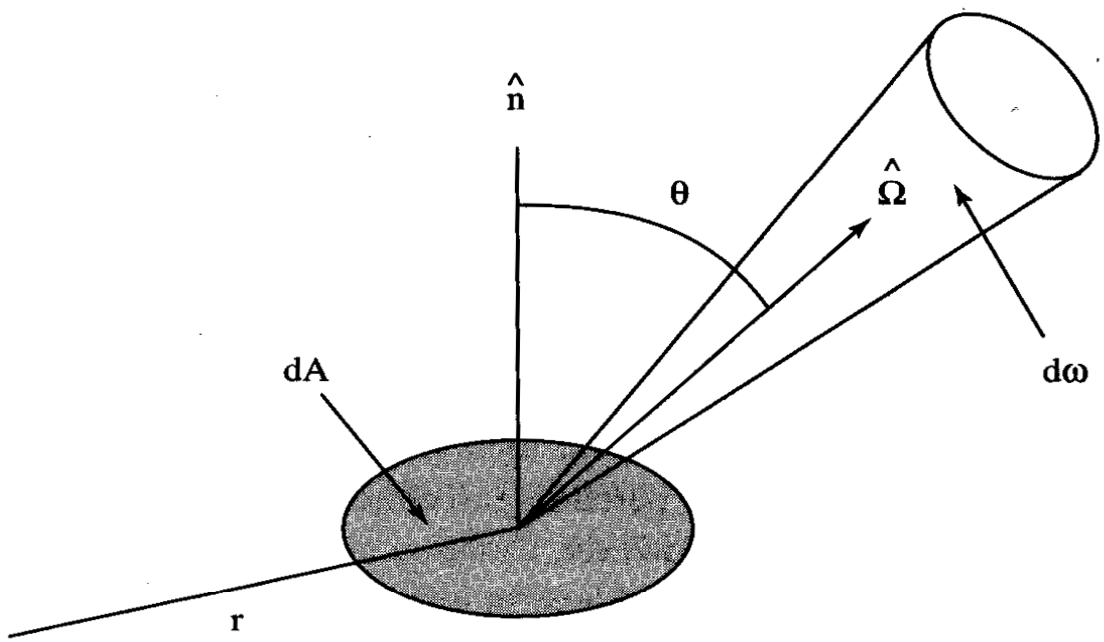


Figure 2.1 The flow of radiative energy carried by a beam in the direction $\hat{\Omega}$ through the transparent surface element dA. The flow direction $\hat{\Omega}$ is at an angle θ with respect to the surface normal $\hat{n}$ ($\cos\theta = \hat{n} \cdot \hat{\Omega}$).

Radiative Flux / Irradiance

- Radiative energy flow (power) per unit area within a small spectral interval $d\nu$ is called the spectral flux

$$F_\nu = \frac{d^3E}{dAdtd\nu}\left(W.m^{-2}.Hz^{-1}\right)$$

- We define two positive energy flows in two separate hemispheres

$$F^+{}_\nu = \frac{d^3E^+}{dAdtd\,\nu}$$

$$F_\nu^- = \frac{d^3E^-}{dAdtd\,\nu}$$

Radiative Flux / Irradiance

- Net energy flow in the +ve direction is

$$d^3E = d^3E^+ - d^3E^-$$

- The net flux is also written in a similar manner

$$F_\nu = F_\nu^+ - F_\nu^-$$

- Summing over all frequencies we obtain the net flux or net irradiance

$$F = \int_0^\infty d\nu F_\nu \quad \left(W.m^{-2}\right)$$

Spectral Intensity

- Defined as

$$I_\nu = \frac{d^4E}{\cos\theta \, dA \, dt \, d\omega \, d\nu}$$

The spectral intensity is the energy per unit area, per unit solid angle, per unit frequency and per unit time. $cos\theta.dA$ is the projection of the surface element in the direction of the beam.

Intensity is a scalar quantity and is always positive.

Flux and intensity

- We can rewrite the equation for I_ν

$$d^4E = I_\nu \cos\theta \, dAdt \; d\omega \, d\nu$$

- The rate at which energy flows into each hemisphere is obtained by integrating the separate energy flows

$$d^3E^+ = \int_+ d^4E^+ \quad , \quad d^3E^- = \int_- d^4E^-$$

Flux and Intensity

- We can now define expressions for the half-range flux

$$F_\nu^+ = \frac{d^3E^+}{dAdtd\nu} = \int_+ d\omega \cos\theta \, I_\nu$$

$$F_\nu^- = \frac{d^3E^-}{dAdtd\nu} = -\int_- d\omega \cos\theta \, I_\nu$$

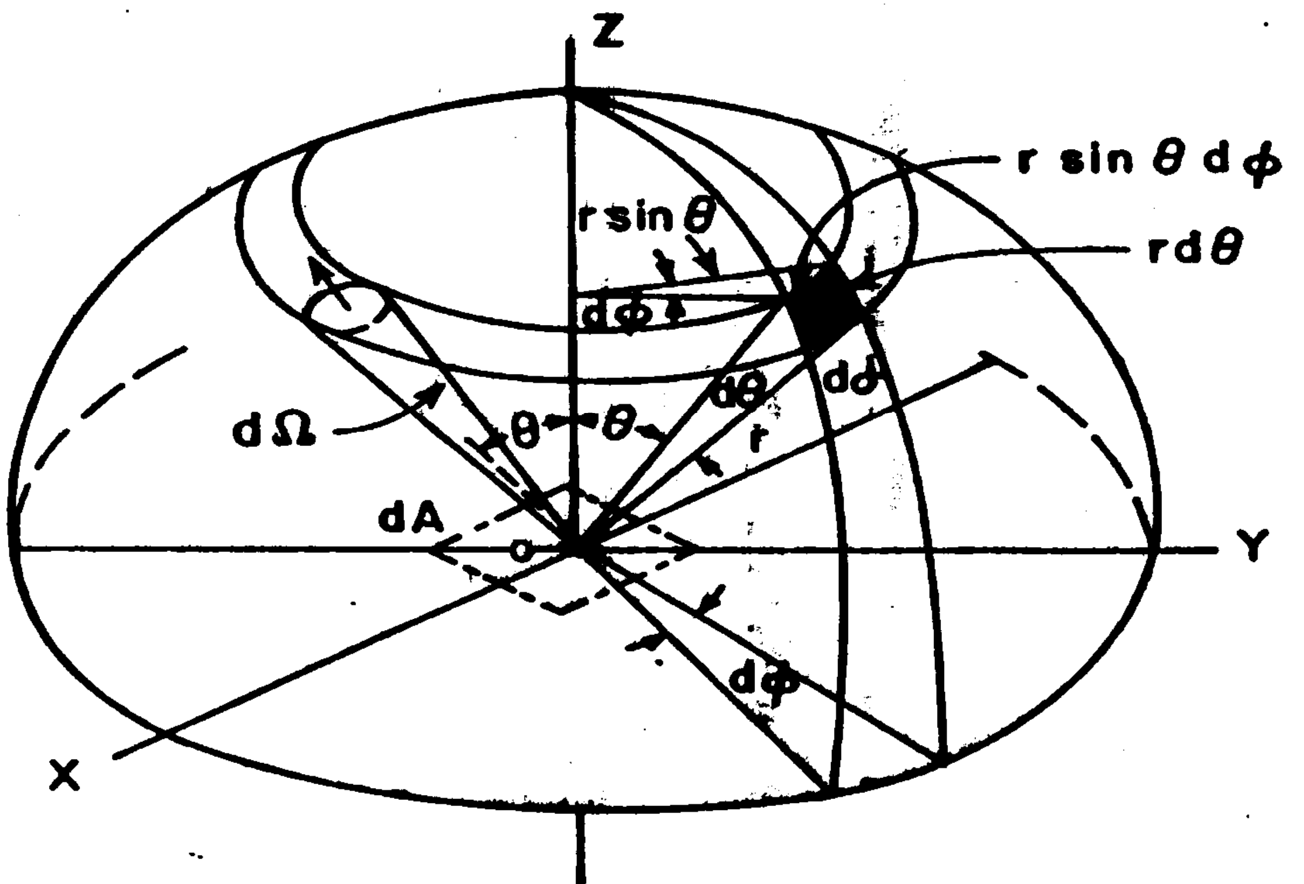
- Where the spectral net flux is given by

$$F_\nu = F_\nu^+ - F_\nu^- = \int_{4\kappa} d\omega \cos\theta \, I_\nu \quad \left[W.m^{-2}.Hz^{-1}\right]$$

Polar Coordinate system

- Each point on the surface of a sphere can be represented by three coordinates
- The distance of the point from the origin, r
- The angle in the xz plane, θ, known as the polar angle.
- The angle in the xy plane, Φ, known as the azimuthal angle

Polar diagram



Polar coordinate system

- The area bounded by dθ and dΦ has dimensions of r.dθ and r.sin θ. dΦ and the solid angle associated with this area is

$$d\omega = \frac{A}{r^2} = \frac{r.d\theta.r\sin\theta.d\phi}{r^2} = d\phi.\sin\theta d\theta$$

Average intensity and energy density

- Averaging the directionally dependent intensity over all directions gives the average Intensity

$$\bar{I}_{\nu}(r) = \frac{1}{4\pi} \oint_{4\pi} d\omega I_{\nu}(r.\overline{\Omega}) \quad \left\{W.m^{-2}.Hz^{-1}\right\}$$

- The energy density is given the symbol U_{ν} , where

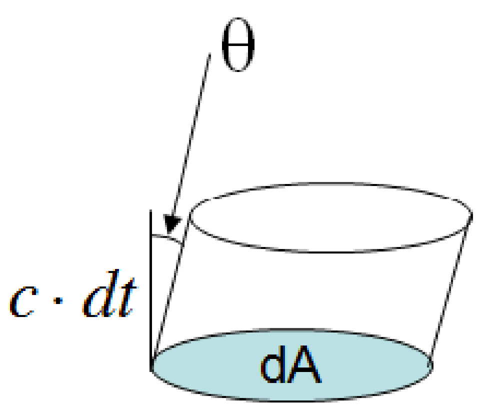
$$dU_{\nu} = \frac{d^{4}E}{dVd\nu} = \frac{I_{\nu}\cos\theta dAdtd\nu d\omega}{dA\cos\theta\, cdtd\nu} = \frac{I_{\nu}}{c} d\omega$$

Actinic flux, Energy density

Radiance averaged over all directions: $\bar{I}_\nu(\mathbf{r}) = \frac{1}{4\pi}\int_{4\pi} I_\nu \cdot d\omega$ **r**: location vector

Actinic flux used in photochemistry is $4\pi\bar{I}_\nu$

Energy density (used in deriving Planck function): U_ν
radiative energy that resides in a unit volume



$$U_\nu = \frac{d^4E}{dV \cdot d\nu} = \frac{I_\nu \cdot \cos\theta \cdot dA \cdot dt \cdot d\omega \cdot d\nu}{\cos\theta \cdot dA \cdot dt \cdot d\nu} = \frac{I_\nu}{c}d\omega$$

Integrating over all directions:

$$U_\nu = \int_{4\pi} dU_\nu = \frac{1}{c}\int_{4\pi} I_\nu \cdot d\omega = \frac{4\pi}{c}\bar{I}_\nu$$

Energy density

- The energy density in the vicinity of a collection of incoherent beams traveling in all directions is given by

$$U_\nu = \int_{4\pi} d\,U_\nu = \frac{1}{c}\int_{4\pi} d\omega\, I_\nu = \frac{4\pi}{c} I_\nu \quad (J.m^{-3}.Hz^{-1})$$

The total energy density is the sum over all frequencies

$$U = \int_0^\infty d\nu U_\nu \qquad (J.m^{-2})$$

Isotropic distribution

- Assume that the spectral density is independent of direction.
- We have previously defined the flux in either hemisphere as

$$
\begin{aligned}
F_\nu &= \int d\omega \cos\theta \, I_\nu \\
&= \iint d\varphi \cos\theta \, I_\nu \sin\theta \, d\theta \\
&= \iint d\varphi \, I_\nu \sin\theta \, d(\sin\theta) \\
&= I_\nu 2\pi \left[\frac{\sin^2\theta}{2} \right]_0^{\pi/2} \\
&= \pi I_\nu
\end{aligned}
$$

Isotropic distribution

- Note that

$$F_\nu = F_\nu^+ - F_\nu^- = 0$$

$$U_\nu = \frac{4\pi I_\nu}{c}$$

Hemispherically Isotropic Distribution

- This situation is the essence of the two stream approximation - to be discussed later.
- Let I^+ denote the value of the constant intensity in the positive hemisphere and I^- that in the negative hemisphere.
- For a slab medium we have replaced an angular distribution of intensity with two average values, one for each hemisphere.
- Although a first sight this may seem inaccurate, for some radiative transfer calculations it is surprisingly accurate.

$$F_\nu = I_\nu^+ \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \sin\theta \cos\theta + I_\nu^- \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} d\theta \sin\theta \cos\theta$$

$$= \pi\left(I_\nu^+ - I_\nu^-\right)$$

$$U_\nu = \frac{I_\nu^+}{c} \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \sin\theta + \frac{I_\nu^-}{c} \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} d\theta \sin\theta$$

$$= \frac{2\pi}{c}\left(I_\nu^+ + I_\nu^-\right)$$